

A Self Study Guide for **Statics in Engineering Mechanics**

(For Under-Graduate Engineering Students)

by

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PREFACE

I must thank CMD of MADE EASY Group, **Mr. B. Singh** for providing me an opportunity to reach out to the Student Community at large through my present book “**A Self Study Guide for Statics in Engineering Mechanics**”. Students may be benefitted from my 60 years of teaching, research and publications through this book.

This book is an initiative to help the needy and mediocre students to self study the subject of **Statics** at home and build their concepts and prepare for examinations with confidence.

Questions in the book have been designed on the pattern of questions that are being asked in university examinations and competitive examinations of UPSC/GATE/PSUs.

The book has been thoroughly reviewed and questions from competitive examinations for the last two years have been added in the book.

Further improvements in the book will be made after getting the response from the students.

Any error in printing or calculations pointed out by the reader will be acknowledged with thanks by the author.

Dr. U. C. Jindal
Author

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01

CHAPTER



Fundamentals of Engineering Mechanics

Mechanics is a branch of physical science, dealing with bodies subjected to external forces (force is caused by external mechanical disturbances). A force causes a body to deform under static condition (or in static) can cause a body to move with acceleration (in dynamics), a moment causes a static body to bend (statics) or a moving body to move with same velocity (in dynamics).

Application of the principles of science of mechanics to practical engineering problem (as a machine, a structure, a train, an aeroplane) is known as **Engineering Mechanics**.

Principles of mechanics are used in the state of :

- (a) strength of material (static)
- (b) stability of structure (static)
- (c) vibration (dynamic)
- (d) rotation (dynamic)
- (e) engine performance (dynamics)

Mechanics is the oldest branch of physical science as :

- 1. Archimedes in 200 BC, gave the principle of buoyancy (floating in liquid), lever (used in machine).
- 2. Sir Issac Newton (1642-1727) gave the principle of gravitational attraction.
- 3. Einstein gave the theory of relativity.

1.1 Basics of Primary Dimensions

Basic dimensions are independent of other dimensions. Dimensions which are developed in terms of basic dimension are known as **secondary dimensions**. There are 3 basic dimension :

- 1. Length
- 2. Time
- 3. Mass

Length is a concept which describes the size of an object quantitatively (as area = length \times breadth, volume = $l \times b \times h$, all are in the unit of length). In SI unit, **metre** is the unit of length. A **straight line** stretched on a metal bar kept at uniform thermal and physical conditions, serves as a simple **invariant** standard of length example for 1 metre, kept at Sèvres France.

Time is a concept for ordering the flow of events. Rotation of earth in one day (24×3600 seconds) is a measure of time. In SI units, a **second** is a unit of time (used in dynamics).

Mass is a property of matter.

In SI system, **kilogram** is used as a measure of mass. The kilogram is measured in terms of response of a body to a **mechanical disturbance**.

SI Unit (System International)

Symbol for length, L

Symbol for time, t

Symbol for mass, M

Figure 1.1 shows a body of mass M , suspended at the end of spring of stiffness, k .

Mg = weight of the mass

g = acceleration due to gravity

δ = deflection in spring due to weight, Mg

then,

$$\delta = \frac{Mg}{k}$$

or,

$$\text{Mass, } M = \frac{\delta k}{g}, \text{ this is how the mass can be measured.}$$

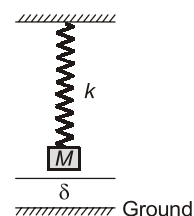


Figure : 1.1

1.2 Secondary Dimensions

When a physical characteristic is defined in terms of basic dimension L , t , M , then such a characteristic is known as **Secondary Dimension**.

Velocity-distance covered by a body in a unit time (distance in terms of length, L)

$$\text{Velocity} = \frac{L}{t} = Lt^{-1}$$

$$\text{Similarly, Acceleration/deceleration} = \frac{L}{t^2} = Lt^{-2}$$

In SI units, the basic dimensions or basic units are :

Basic Units	Unit Symbol	Dimension	Physical Quantity
Kilogram	kg	M	Mass
Metre	m	L	Length
Second	S	t	Time
Kelvin	K	T	Temperature

Supplementary Units :

Basic Units	Unit Symbol	Dimension	Physical Quantity
Radian	rad	–	plane angle
Steradian	Sr	–	solid angle

Secondary Units :

Derived Unit	Unit Symbol	Physical Quantity
Newton	$N = \text{kgm/s}^2$	Force
Joule	$J = \text{Nm} = \text{kgm}^2/\text{s}^2$	Energy, watt, heat
Watt	$W = \text{J/s} = \text{kgm}^2/\text{s}^3$	Power
Pascal	$\text{Pa} = \text{N/m}^2 = \text{Kgm/s}^2\text{m}^2 = \text{kg/ms}^2$	Pressure, Stress
Hertz	$\text{Hz} = \text{s}^{-1}$	Frequency

1.3 Dimensional Homogeneity

Law of dimensional homogeneity states that “the basic equation representing a physical phenomenon must be valid for all system. of units.”

These are following system of units :

1. FPS (foot, pound, second)
2. MKS (metre, kilogram, second)
3. SI (metre, newton, second)

Note that unit of time is **second** in all the systems.

Example of **simple pendulum**.

Time period of a simple pendulum (for making one complete oscillation),

M to B →

B to A ←

A to M →

This time period, T ,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where,

L = length of pendulum OA

g = acceleration due to gravity

is valid in all system of units. The resultant unit of terms on both sides of the equation is second (s).

Equation of a straight line : Point B in straight line

$$y = mx + C$$

m = Slope

C = Intercept C

Dimension are of length (L) on both the sides.

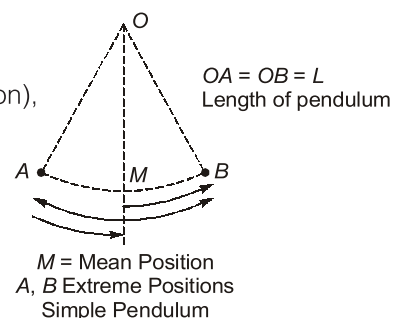


Figure : 1.2

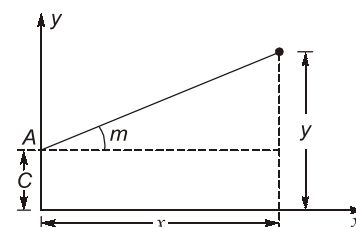


Figure : 1.3

1.4 Relation between Force and Mass

Mechanical disturbance causes a force = Mass × Acceleration

$$= M \times a$$

$$= (M) \left(\frac{L}{t^2} \right) = MLt^{-2}$$

Unit of force,

$$F = MLt^{-2}$$

or, Acceleration,

$$a = Lt^{-2} = \frac{F}{M} = \frac{\text{Force}}{\text{Mass}}$$

In SI units :

Mass,

$$M = \frac{\text{Force (Newton)}}{\text{Acceleration} \left(\frac{m}{s^2} \right)}$$

$$= \frac{N}{\left(\frac{m}{s^2} \right)}$$

$$= Nm^{-1} s^2$$

Weight,

$$W = Mg \text{ (Mass} \times \text{Acceleration due to gravity)}$$

or

$$M = \frac{W \text{ (Newton)}}{g(m/s^2)} = Nm^{-1} s^2$$

Mass, M is in kilogram.

A kilogram is the amount of mass that will accelerate by 1 m/s^2 under the action of 1 newton force.

One **kg-force** (kg-f) is the weight of 1 kilogram of mass at the earth's surface, where acceleration due to gravity is $g = 9.81 \text{ m/s}^2$.

$$9.81 \text{ Newton} = 1 \text{ kgf} = 1 \text{ kilogram of force}$$

1.5 Idealization in Mechanics

A body can be a heterogeneous mixture of some constituents, may contain defects as crack, holes, yet it is considered as a **homogeneous matter** for the analysis of forces and moments on it. For example :

- (a) A concrete is a mixture of stone granules, slurry of cement, sand and water, yet it is considered as a homogeneous matter as concrete. Its value of E and σ_c , compressive strength are given.
- (b) A wooden plank may contain fibres bonded by a glucose matter. Yet it is considered as homogeneous.
- (c) Cast iron contains graphite flakes, blow holes yet it is considered as homogeneous with its E and σ_c , ultimate compressive strength.

Hypothetically, continuous distribution of matter called the continuum.

These assumptions are made to simplify the analysis of these bodies in mechanics.

1.6 Rigid Body

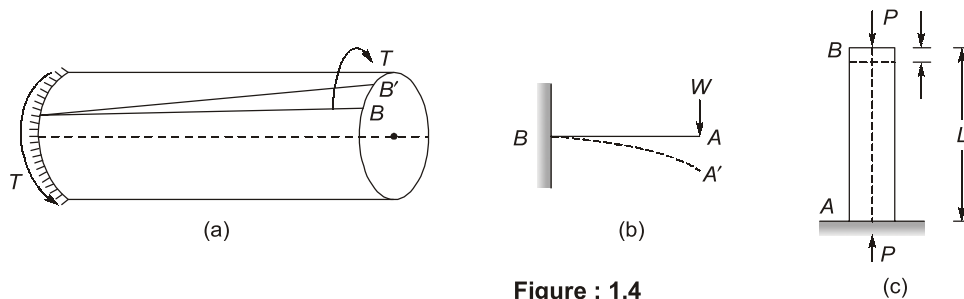


Figure : 1.4

In mechanics, we consider that a body on which forces and moments act, does not change its dimensions and shape. Figure 1.4(a) shows a shaft subjected to a twisting moment T at one end. At the other end there is resisting moment, T in anti-clockwise direction, for equilibrium. But there is twist in the shaft as a line AB drawn parallel to the axis of the shaft takes new position AB' . In mechanics, we consider that line AB remains same as AB , no distortion.

In figure 1.4(b), a cantilever AB subjected to a point load W at end A . Under the action of the load W , cantilever bends, point A comes down to A' and the cantilever is curved as shown. But in statics, BA remains straight after the application of the load W . Refer figure 1.4(c) shows a bar BA , subjected to compressive force P . There is contraction in length δL , due to compressive load. But in statics, $\delta L \rightarrow 0$.

1.7 Point Force

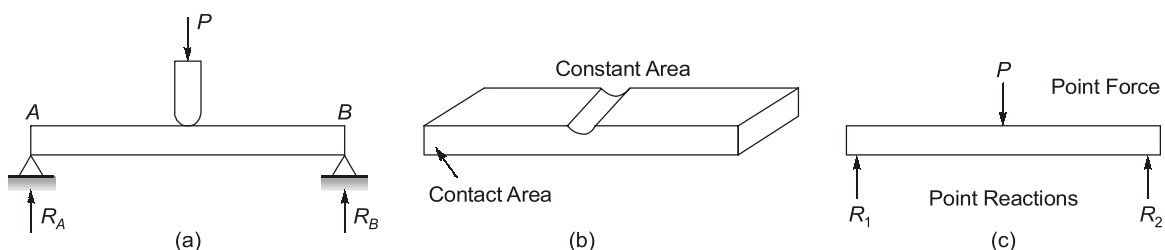


Figure : 1.5

Finite force (force of known magnitude) exerted by one body over another body creates finite areas of contact between two bodies. Figure 1.5(a) shows a beam AB , supported over two wedges. A force P is applied through a central wedge. If this wedge is sharp, it will create V-notch in the beam.

Description : A finite force is acting on line contact or a point contact. Figure 1.5(c) shows a force P at a point and reactions, R_1 and R_2 at two points.

Particle : In Mechanics, dimensions of the size of the mass are not considered but it is considered as a **particle** which has no size. An aeroplane, while considering the effect of thrust is considered only as a particle of given mass, M .

During the analysis of the motion of a body, as train, stone, aeroplane etc. a body is modelled as a particle of given mass, M .

1.8 Scalar and Vector Quantity

In mechanics, there are two types of quantities, i.e.,

- Scalar quantity**, which has magnitude only such as volume of a body, density, speed, energy, mass and time.
- Vector quantity** : In addition to magnitude, the direction is also specified as in the case of displacement, velocity, acceleration, force, moment (force into distance) and momentum (mass \times velocity).

1.9 Vector Classification

Vectors can be classified as : (i) Free vector; (ii) Sliding vector ; (iii) Fixed vector.

- Free Vector** : A free vector is one whose action is **not confined to a unique line in space**. Figure 1.6(a) shows a body moving in space without rotation, then displacement at any point in the body can be taken as vector and this point will define the displacement (direction and magnitude) of every point in the body.

Point P is located on the body, its position vector such as OP , velocity of the point P is V , displacement along the direction of V . The velocity and displacement of any point say A on the body will be the same as of the point P , or any other point in the body.

OP (position vector) is the free vector, the position and direction of any other point on the body are well defined by the position vector.

- Sliding Vector** : A sliding vector is one for which a unique line of action in space is maintained. An external force P acts on the body. Vector can slide along the line of action, CBA as shown, **as long as the point remains on the body**, i.e., force P can act at A , B or C along the line of action ABC . Such vector is also called the **transmissible vector**.

- Fixed Vector** : A **fixed vector** is one for which a unique point of application is given in space and vector occupies a unique position in space. The action of a force on a deformable body is specified by a fixed vector. Figure 1.6(c) shows a bar on which a force $F\downarrow$ acts a point A , which compresses the body and its length is reduced. To maintain equilibrium an equal and opposite force $F\uparrow$ acts upward at the point B , on a plane in contact with ground.

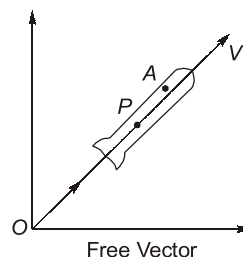


Figure : 1.6 (a)

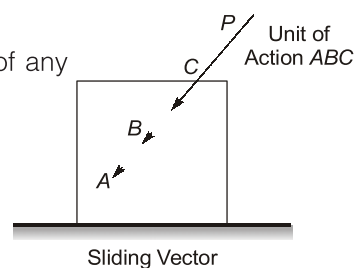


Figure : 1.6(b)

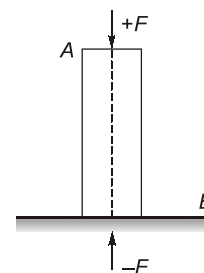


Figure : 1.6(c)

Vector Representation

A vector F is represented by a line segment having the **direction** of the vector and a **arrowhead** shows the source of direction. Length of the line segment represent the magnitude of F (to same scale) as shown in figure 1.6(d).

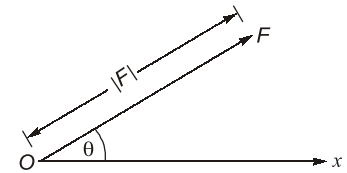


Figure : 1.6(d)

1.10 Equality and Equivalence of Vectors

Two vectors are equal if their magnitude and directions are the same, direction (as shown by angle θ), the $|F_1| = |F_2|$ magnitude same. Though their points of application A and B are different, as shown in figure 1.7(a).

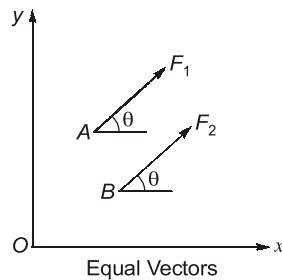


Figure : 1.7 (a)

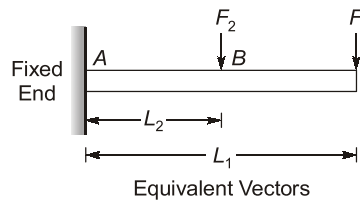


Figure : 1.7(b)

Two vectors are equivalent in a particular capacity if each produces the same effect. Figure 1.7(b) shows two free vectors F_1 at C and F_2 at B , producing same moment about the fixed end A .

i.e., $F_1 \times L_1 = F_2 \times L_2$ (equivalence in capacity of producing moment)

1.11 Gravitational Law of Attraction

This law states that two bodies of masses m_1 and m_2 are mutually attracted by equal and opposite forces F and $-F$ of magnitude $|F|$ given by the formula :

$$|F| = G \frac{m_1 m_2}{r^2}$$

where,

r = distance between two bodies of mass m_1 and m_2

G = universal constant called the constant of gravitation

Say,

m = mass of a body

M = mass of the earth

$F = W$, weight of the body

= pull exerted by the earth

Then,

$$W = \frac{mMG}{r^2}$$

where

$r = R$, radius of the earth

$$W = \frac{mMG}{R^2}$$

$$\frac{W}{m} = \frac{MG}{R^2} = g, \text{ acceleration due to gravity}$$

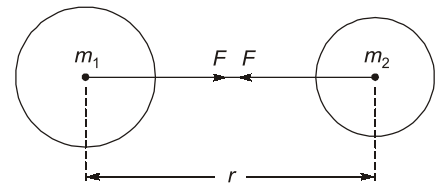


Figure : 1.8

1.12 Fundamental Principles of Mechanics

Following laws are considered as the foundations of mechanics :

1. Newton's first law of motion
2. Newton's second law of motion
3. Newton's third law
4. Gravitational law of attraction
5. Law of parallelogram of forces
6. Principle of transmission of force

1.13 Some Useful Quantities

Speed of light,	$c = 2.997928 \times 10^8$ m/second
Gravitational constant,	$G = 6.670 \times 10^{-11}$ Nm ² /kg ²
Gravitational parameter,	$GM = 3.986 \times 10^{14}$ m ² /s ² where 's' stands for second
Mass of earth,	$M = 5.976 \times 10^{24}$ kg
Mean radius of earth,	$R = 6371$ km
Escape velocity at the surface of earth,	$V_e = 11.2$ km/s

1.14 Useful Multiplication Factors

Tera,	$T = 10^{12}$	Atto,	$a = 10^{-18}$
Giga,	$G = 10^9$	Femto,	$f = 10^{-15}$
Mega,	$M = 10^6$	Pico,	$p = 10^{-12}$
Kilo,	$K = 10^3$	Nano,	$n = 10^{-9}$
Hecto,	$h = 10^2$	Micro,	$\mu = 10^{-6}$
Deka,	$da = 10$	Milli,	$m = 10^{-3}$
		Centi,	$c = 10^{-2}$
		Deci,	$d = 10^{-1}$

1.15 Important Trigonometric Functions

$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta \\ \sin^2 \theta + \cos^2 \theta &= 1 \\ 1 + \tan^2 \theta &= \sec^2 \theta \\ 1 + \cot^2 \theta &= \operatorname{cosec}^2 \theta \\ \sin \theta &= \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots \\ \cos \theta &= 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots\end{aligned}$$

1.16 Useful Integration Formulas

$$1. \int \frac{x dx}{a + bx} = \frac{1}{b^2} [a + bx - a \ln(a + bx)]$$

2. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \cdot \ln \left(\frac{a+x}{a-x} \right)$
3. $\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right]$
4. $\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left(x\sqrt{a^2 - x^2} + a^2 \sin^{-1} \frac{x}{a} \right)$
5. $\int x\sqrt{a^2 - x^2} dx = -\frac{1}{3} \sqrt{(a^2 - x^2)^3}$
6. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a}$
7. $\int \sin^2 \theta d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta$
8. $\int \cos^2 \theta d\theta = \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$
9. $\int \sin^2 \theta d\theta = -\frac{1}{3}\cos \theta (\sin^2 \theta + 2)$
10. $\int \cos^m \theta \sin \theta d\theta = -\frac{\cos^{m+1} \theta}{m+1}$
11. $\int \sin^m \theta \cos \theta d\theta = \frac{\sin^{m+1} \theta}{m+1}$
12. $\int \sin^m \theta d\theta = -\frac{\sin^{m-1} \theta \cdot \cos \theta}{m} + \frac{m-1}{m} \int \sin^{m-2} \theta \cdot d\theta$
13. $\int \theta^2 \sin \theta d\theta = 2\theta \sin \theta - (\theta^2 - 2)\cos \theta$
14. $\int \theta^2 \cos \theta d\theta = 2\theta \cos \theta + (\theta^2 - 2)\sin \theta$
15. $\int \theta \sin^2 \theta d\theta = \frac{1}{4} [\sin \theta (\sin \theta - 2\theta \cos \theta) + \theta^2]$
16. $\int \sin m\theta \cos m\theta d\theta = -\frac{1}{4m} \cos 2m\theta]$
17. $\int \theta \sin \theta d\theta = \sin \theta - \theta \cos \theta$
18. $\int \theta \cos \theta d\theta = \cos \theta + \theta \sin \theta$
19. $\int u dv = uv - \int v du$



02

CHAPTER

Vector Algebra

In chapter 1, we have learnt about scalar and vector quantities. Figure 2.1(a) shows a concrete block of mass 80 kg – a scalar quantity. Figure 2.1(b) shows a force of 80 N, along the direction AB , Magnitude of 80 N, a vector quantity.

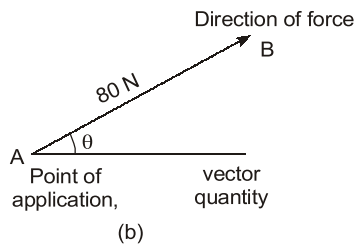
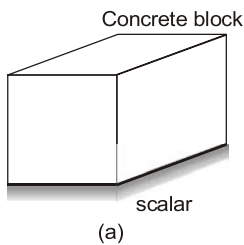


Figure : 2.1

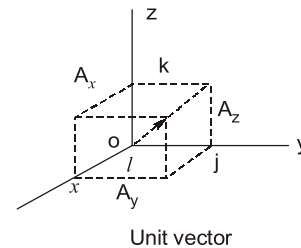


Figure : 2.2

2.1 A Vector – Its Direction Cosines and Magnitude

Let us take vector A , as follows : $A = A_x i + A_y j + A_z k$, where, A_x, A_y, A_z are components of vector along x, y, z co-ordinates respectively. i, j, k are unit vectors along x, y and z direction as shown in figure 2.2.

Magnitude of the Vector : $|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Direction considers are $l = \frac{A_x}{|A|}; m = \frac{A_y}{|A|}, n = \frac{A_z}{|A|}$

Let us take

So that,

$$A_x = +6, A_y = +7, A_z = -4$$

$$A = 6i + 7j - 4k$$

...(1)

Question 2.1

Draw x, y, z coordinates.

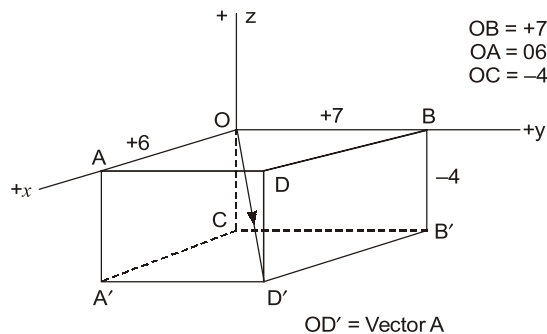


Figure : 2.3

Take

 $OA = +6$, along +ve x -direction $OB = +7$, along +ve y -direction $OC = -4$, along -ve z -directionComplete the parallelopiped $OBCD$, $CA'D'B'$, then O to D' is the magnitude of the vector A .

Solution:

$$\begin{aligned}
 |A| &= \sqrt{A_x^2 + A_y^2 + A_z^2} \\
 &= \sqrt{6^2 + 7^2 + (-4)^2} = \sqrt{36 + 49 + 16} = \sqrt{101} \\
 &= 10.05 \text{ units}
 \end{aligned}$$

Direction cosines :

$$l = \frac{A_x}{|A|} = \frac{6}{10.05} = +0.5970, \alpha = +53.34^\circ = \cos^{-1} 0.5970$$

$$m = \frac{A_y}{|A|} = \frac{7}{10.05} = +0.6965, \beta = +45.85^\circ$$

$$n = \frac{A_z}{|A|} = \frac{-4}{10.05} = -0.398, \gamma = -66.55^\circ$$

$$\begin{aligned}
 l^2 + m^2 + n^2 &= 0.3564 + 0.4851 + 0.1584 \\
 &= 0.999904 \simeq 1
 \end{aligned}$$

Practice Q.2.1 Vector : $A = +8i - 6j + 4k$ Show the vector on x - y - z coordinates. Determine its magnitude and direction cosine.Ans. [$|A| = 10.77$, $l = +0.7428$, $m = -0.5570$, $n = +0.3714$, $\alpha = 42^\circ$, $\beta = -56.15^\circ$, $\gamma = 68.19^\circ$]**Practice Q.2.2** Vector : $B = +7i + 7j - 7k$ Show the vector on x - y - z coordinates. What is the magnitude of B and its direction cosines.Ans. [$|B| = 12.124$, $l = +0.577$, $m = +0.577$, $n = -0.577$, $\alpha = 54.76^\circ$, $\beta = 54.76^\circ$, $\gamma = -54.76^\circ$]

2.2 Addition and Subtraction of Vectors

Vectors can be added or subtracted by using the law of parallelogram of forces (vectors). Figure 2.4(a) shows a vector A and vector B , Resultant of $A + B = R$, R is obtained by drawing line parallel to vectors A and B , and joining the intersecting points at C . Then OC represents vector, $C = A + B$.

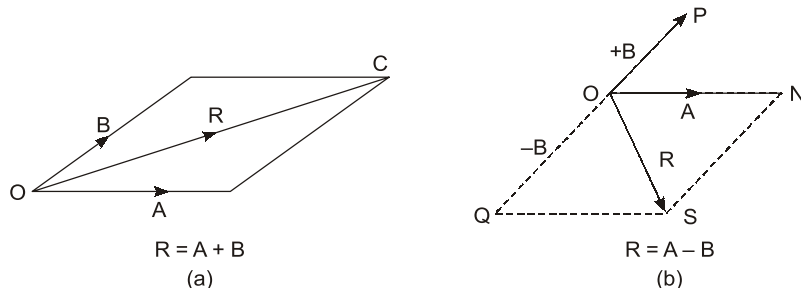


Figure : 2.4

Figure 2.4(b) shows vectors A and B . To obtain $C = A - B$, let us draw vector $OQ = -B$, where $OP = +B$. Drawing line parallel to vector OA (ON) and parallel to OQ ($-B$). Completing the parallelogram $ONSQ$, join the line OS , represent with $R = A - B$.

Question 2.2 Figure 2.5 shows a particle O , subjected to force of 8 kN along x -axis and 6 kN in a direction at an angle of 45° to x -axis. Determine the resultant for the force.

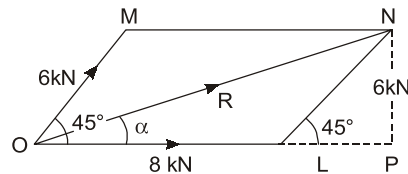


Figure : 2.5

Solution:

Complete the parallelogram $OLMN$, then $ON = R$, resultant

$$\begin{aligned} R &= \sqrt{(OL + LP)^2 + (NP)^2} = \sqrt{(8 + 6\cos 45^\circ)^2 + (6\sin 45^\circ)^2} \\ &= \sqrt{(8 + 4.242)^2 + (6 \times 0.707)^2} = \sqrt{(12.242)^2 + (4.242)^2} \\ &= \sqrt{150 + 18} = 12.96 \text{ N} \end{aligned}$$

Angle α :

$$\cos^{-1} \frac{12.242}{12.96} = \cos^{-1} 0.945 = 19.16^\circ$$

Vector Method :

$$A = 8i \text{ N}$$

$$B = 3\sqrt{2}i + 3\sqrt{2}j$$

$$R = A + B$$

$$= (8 + 3\sqrt{2})i + 3\sqrt{2}j = 12.242i + 4.242j$$

$$R = \sqrt{(12.242)^2 + (4.242)^2} = \sqrt{150 + 18} = 12.96 \text{ N}$$

$$\cos \alpha = \frac{12.242}{12.96} = 0.9446$$

$$\alpha = 12.96^\circ$$

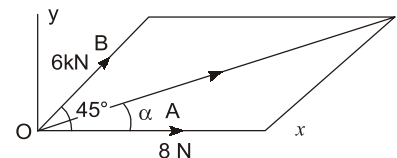


Figure : 2.6

Note that vector method is very simple, less time consuming and there is no need even to draw the figure (of parallelogram).

Practice Q.2.3

$$\theta = \tan^{-1} \frac{4}{6} = 33.70^\circ$$

Particle O is subjected to forces $A = 10 \text{ N}$ along x -direction, $B = 7.211 \text{ N}$ at angle $\theta = 33.7^\circ$ with x -axis. Determine $A + B$ by parallelogram method. What are its direction cosines.

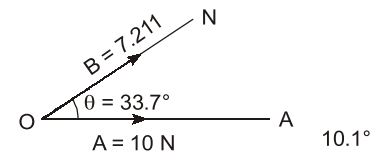


Figure : 2.7

Ans. $[R = 16.49 \text{ N}, \alpha = 14.06^\circ, B = 75.94^\circ, l = 0.970, m = 0.2456]$

2.3 Resolution of a Vector

1. A vector can be resolved in any two coplanar components in any two desired directions. In case of two dimensional resolution, taking the vector as a diagonal and completing a parallelogram from two ends of the vector, drawing lines parallel to the two desired directions as shown in figure 2.8.

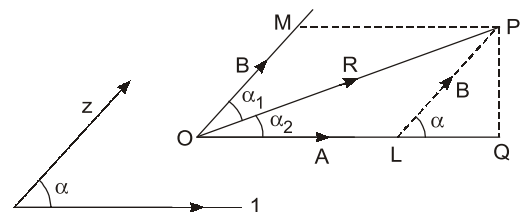


Figure : 2.8

Vector R is represented by positive vector OP . Along direction 1 and 2, we have to find components of R in these two directions.

Draw OL parallel to direction 1. OM parallel to direction 2, complete the parallelogram as shown in figure 2.8.

Then, complete along direction 1, $A = OL$

Complete along direction 2, $B = OM$

From the figure, $R \sin \alpha_2 = B \sin \alpha = PQ$... (1)

$$R \cos \alpha_2 - B \cos \alpha = OQ - LQ = OL = A$$

Angle α_2 can be measured from the parallelogram in figure 2.8.

2. Resolution of a vector along x - y - z direction. Figure shows a vector R , inclined at angles α , β and γ with respect to x -axis, y -axis and z -axis respectively.

$OM = R$, magnitude and direction of vector R . α , β , γ are angles of R with respect to x -axis, y -axis and z -axis respectively.

Then, components of R ,

$$\begin{aligned} R_x &= R \cos \alpha, & \cos \alpha &= \cos(A, x) \\ R_y &= R \cos \beta, & \cos \beta &= \cos(A, y) \\ R_z &= R \cos \gamma, & \cos \gamma &= \cos(A, z) \end{aligned}$$

Note that : $AN = A_x$ (+ve x direction)

$NL = A_y$ (+ve y direction)

$LM = A_z$ (+ve z direction)

So, $A = A_x i + A_y j + A_z k$

where i , j , k are unit vectors along x , y , z directions.

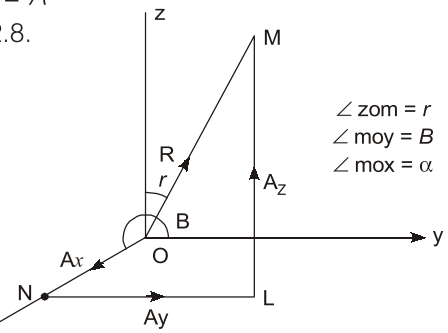


Figure : 2.9

Question 2.3 A vector $OP = 100$ N force is represented along OP , where coordinates of the point P are $(-6, +8, +10)$. Express force 10 N in terms of unit vectors i , j and k .

(Components of the force 100 N, along x , y , z direction)

Solution:

Direction,

$$OP = -6i + 8j + 10k$$

$$|OP| = \sqrt{36 + 64 + 100} = 14.142$$

Direction cosine,

$$\cos \alpha = \frac{-6}{14.142} = -0.4243 = l$$

$$\cos \beta = \frac{+8}{14.142} = +0.5657 = m$$

$$\cos \gamma = \frac{+10}{14.142} = +0.707 = n$$

Force of 100 N can be expressed as :

$$100 = 100[l i + m j + n k]$$

$$100 = [-0.4243i + 0.5657j + 0.707k]$$

From

$$100 \text{ N} = -42.43i + 56.57j + 70.7k \text{ N}$$

where -42.43 , $+56.57$, $+70.7$ are scalar components.

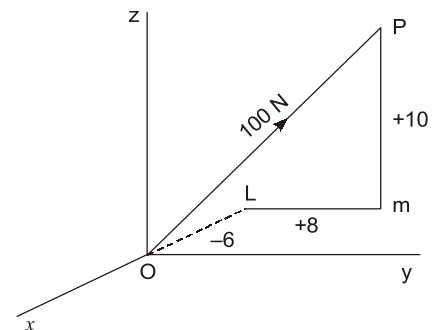


Figure : 2.10

Question 2.4 Figure 2.11 shows a block lying on inclined plane OA , with angle of inclination of 30° with the horizontal. A force $P = 150$ N acts on the block at an angle of 30° with the plane OA . Determine the components of $P = 150$ N along directions OA and OY (y -axis)

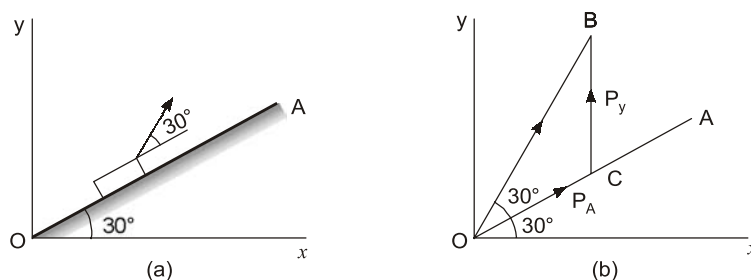


Figure : 2.11

Solution:

OP is inclined at angle $30^\circ + 30^\circ = 60^\circ$ with the x -axis as shown by direction OB .

From B draw a line parallel to y -axis intersecting the plane OA at C .

$OC = P_A =$ components along the plane OA

$BC = P_y =$ components of the force along the y -direction

From $\triangle OCB$,

$$P_A \cos 30^\circ = OB \cos 60^\circ$$

$$P_A = OB \times \frac{\cos 60^\circ}{\cos 30^\circ} = OB \times \frac{0.5}{0.866} = 0.577 OB$$

$$\begin{aligned} P_y &= OB \sin 60^\circ - P_A \sin 30^\circ \\ &= OB \times 0.866 - 0.577 \times OB \times 0.5 \\ &= 0.577 OB \end{aligned}$$

But

$$OB = 150 \text{ N}$$

So,

$$P_A = 0.577 \times 150 = 86.55 \text{ N}$$

$$P_y = 0.577 \times 150 = 86.55 \text{ N}$$

Practice Q.2.4 Figure 2.12 shows a block lying on an inclined plane OA , with inclination of 15° with the horizontal. A force of $P = 100$ acts on the block on angle of 25° . Determine components of P along plane OA and y -axis.

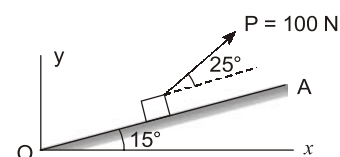


Figure : 2.12

Ans. [79.3 N, 47.756 N]

2.4 Scalar Product or Dot Product of Vectors

There are two vectors A and B .

Scalar product of A and $B = A \cdot B$

$|A|$ = magnitude of vector A

$|B|$ = magnitude of vector B

α = Smallest angle between the vectors A and B as shown in figure 2.12.

Then,

$$A \cdot B = |A| |B| \cos \alpha$$

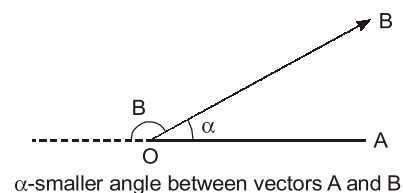


Figure : 2.13

Let us say, Vector A represents the force on particle O .
 Vector B represents the displacement vector for particle O .
 Work done,

$$U = A \cos \alpha \times B \\ = |A| |B| \cos \alpha$$

Work is a scalar quantity.

Let us take,

$$A = A_x i + A_y j + A_z k \\ B = B_x i + B_y j + B_z k \\ A \cdot B = A_x B_x + A_y B_y + A_z B_z$$

Note :

$$i \cdot i = \cos \alpha = \cos 0 = 1$$

(both acting x -axis)

Similarly,

$$j \cdot j = k \cdot k = \cos \alpha = \cos 0^\circ = 1$$

(Angle $\alpha = 0$)

And

$$ij = jk = ki = \cos 90^\circ = 0, \text{ as shown in figure 2.14.}$$

We have already learnt that rectangular components of a vector

$$C = C_x i + C_y j + C_z k$$

are C_x, C_y, C_z along x, y, z direction.

Moreover,

$$|C| = \sqrt{C_x^2 + C_y^2 + C_z^2}$$

Direction cosines are

$$\cos \alpha = l = \frac{C_x}{|C|}$$

$$\cos \beta = m = \frac{C_y}{|C|}$$

$$\cos \gamma = n = \frac{C_z}{|C|}$$

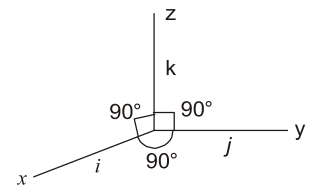


Figure : 2.14

Question 2.5 Given the vectors $A = 10i - 15j$, $B = -5i + 3j$. What vector C gives $C \cdot A = -30$, $C \cdot B = -3$. Determine vector C .

Solution:

Say,

$$\text{Vector } C = C_x i + C_y j$$

$$C \cdot A = 10C_x - 15C_y = -30 \quad \dots(1)$$

$$C \cdot B = -5C_x + 3C_y = -3$$

or $-10C_x + 6C_y = -6 \quad \dots(2)$

From equations (1) and (2)

$$-9C_y = -36$$

$$C_y = +4 \quad \dots(3)$$

$$10C_x = -30 + 15C_y = -30 + 15 \times 4 = +30$$

$$C_x = 3$$

So,

$$\text{Vector, } C = 3i + 4j$$

Practice Q.2.5 Given vectors $A = 10i + 15j$, $B = -5i - 3j$. What vector C , gives $C \cdot A = +5$, $C \cdot B = -16$.

Ans. $[C = 5i - 3j]$

Question 2.6 Given vectors : $A = 8i + 12j - 3k$,

$$B = 5j + 3k$$

What is $A \cdot B$? What is $\cos(A, B)$. What is the projection of B over A .

Solution:

$$A = 8i + 12j - 3k$$

$$B = 5j + 3k$$

$$A \cdot B = 60 - 9 = +51$$

$$|A| = \sqrt{64 + 144 + 9} = 14.73$$

$$|B| = \sqrt{25 + 9} = 5.83$$

$$|A| |B| \cos \alpha = 51$$

$$14.73 \times 5.83 \times \cos \alpha = 51$$

$$\cos \alpha = \frac{51}{85.88} = 0.5938$$

$$\alpha = \cos^{-1} 0.5938 = 53.57^\circ$$

$$\text{Projection of } B \text{ along } A = 5.83 \cos \alpha$$

$$= 5.83 \times 0.5938$$

$$= 3.46$$

$$|A| |B| \cos \alpha = 14.73 \times 3.46 = 51$$

Practice Q.2.6

Vectors A , B and C are given as follows : $A = 10j + 3k$, $B = 20i + 3j$, $C = -5j$.

Compute : $C(A \cdot C) + B$.

Ans. $[20i + 253j]$

2.5 Unit Vector

Let us take vector

$$A = A_x i + A_y j + A_z k$$

$$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\frac{A_x}{|A|} = l, \quad \frac{A_y}{|A|} = m, \quad \frac{A_z}{|A|} = n$$

where direction cosines are l , m , n .

Unit vector,

$$\hat{r} = \frac{A}{|A|}$$

$$\hat{r} = li + mj + nk$$

...(1)

where,

$$\hat{r} = \text{unit vector}$$

Figure 2.15 shows a unit vector \hat{r} , with its rectangular components r_x , r_y , r_z .

So that,

$$\hat{r} = r_x i + r_y j + r_z k$$

...(2)

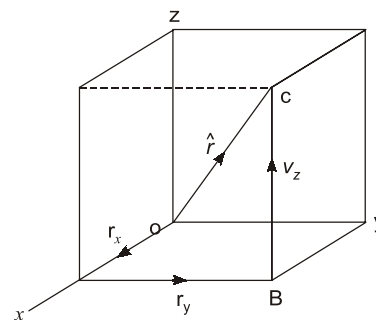


Figure : 2.15

Question 2.7 A force is given by a vector $C = 10i + Aj + Bk$. What must be A and B to give a rectangular component of 12 N in the direction $\hat{r}_1 = 0.4i + 0.6j + 0.693k$, as well as component of 21 N in the direction $\hat{r}_2 = 0.3i + 0.6j + 0.742k$.

Solution:

$$C \cdot \hat{r}_1 = 10 \times 0.4 + 0.6A + 0.693B$$

$$= 4 + 0.6A + 0.693B = 12 \text{ N} \quad \dots(1)$$

$$C \cdot \hat{r}_2 = 3 + 0.6A + 0.742B = 21 \text{ N} \quad \dots(2)$$